AP Calculus BC - Summer Assignment 2021

Please complete the 2020 AP Calculus AB Practice Exam (by Patrick Cox) to review the topics covered in Honors Calculus. You will need to omit 3 problems as they have not yet been covered (they are highlighted and have the word OMIT next to them!). The answers are provided at the end of the packet - please be sure to check your answers!

In addition, there are 4 10-Point Problems attached to the Summer Assignment. These are to be completed and handed in on the first day of class. You must show all work to receive full credit.

Good luck!!

AP Calculus BC
JBHA - Strick
Summer Assignment - 10-Point Problems

Name:________________________

Date:________________________

Part I - No Calculator

1. The velocity of a particle moving on the x-axis is given by $v(t) = 12t^2 - 36t + 15$ for $t \geq 0$. At $t = 1$, the particle is at the origin.

(a) Find the position $s(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of t for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. Given the curve $2+xy=y^2$

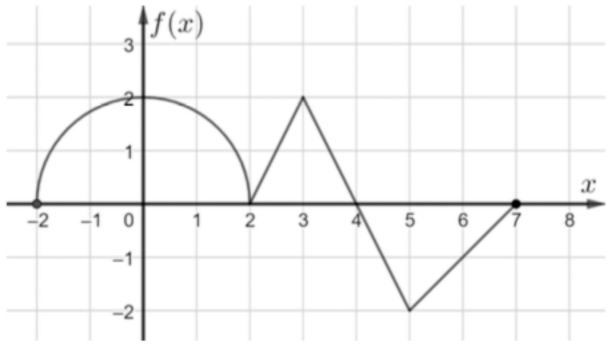
(a) Show that $\frac{dy}{dx}=\frac{y}{2y-x}$

(b) Find all points on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be the functions of time t that are related by the equation $2+xy=y^2$. At time $t=5$, the value of y is 3 and $\frac{dy}{dx}=6$. Find the value of $\frac{dx}{dt}$ at time $t=5$.

Part II - Calculator



3. The function f is continuous on the interval $[-2,7]$ and consists of three line segments and a semi-circle as shown in the figure above. The function g is defined by $g(x)=\int_{-2}^{x^2} f(t)\,dt$.

(a) Find $g(2)$, $g'(2)$ and $g''(2)$.

(b) Let $h(x)=f(5x-9)$. Find $h'(3)$.

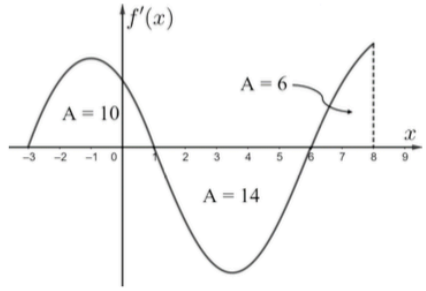
(c) Evaluate $\int_{-1}^{0}\left[f'(3-2x)-4\right]dx$.

t seconds	0	1	4	6
$P(t)$ people per second	8	3	5	10

For $0 \le t \le 6$ seconds, people enter a school at the rate $P(t)$, measured in people per second.

(a) Approximate $P'(5)$. Using correct units, interpret the meaning of $P'(5)$ in the context of the problem.

(b) Use a left Riemann sum with the three subintervals indicated by the table above to approximate $\int_0^6 P(t)\,dt$.



x	1	4	6	9
$g(x)$	3	1	0	-1
$g'(x)$	2	0	1	3

4. A portion of the graph of f ', the derivative of the twice differentiable function f, is shown in the figure above. The areas of the regions bounded by the graph of f ' and the x-axis are labeled. It is known that $f(1) = -2$.

The function g is twice differentiable. Selected values of g and g' are shown in the table above.

(a) Find all values of x in the open interval $-3 < x < 8$ for which the graph of f has a horizontal tangent line. For each value of x, determine whether f has a relative minimum, relative maximum or neither a minimum nor a maximum at the x value. Justify your answers.

(b) Find the minimum value of f on the closed interval $[-3, 8]$. Justify your answer.

(c) Let $h(x) = \frac{e^{g(x)}}{3x}$. Find h'(6).

(d) Is there a time c, $1 < c < 9$, such that $g'(c) = -\frac{1}{2}$? Give a reason for your answer.

(e) Evaluate $\int_1^4 [g(x)]^2 g'(x)\,dx$.

2020 AP Calculus AB Practice Exam

By: Patrick Cox

Original non-secure materials written based on previous secure multiple choice and FRQ questions from the past three years. I wrote this as a way for my students to have access to multiple choice and FRQ since secure materials can't be used outside of class.

Feel free to use in your class, post to the internet/classroom, you will find the answer key to the multiple choice and FRQ at the end. Below, you can find which problems can be answered after each unit in the CED (although questions definitely can span across multiple units in the CED). I do not work for Collegeboard so these categorizations are to the best of my knowledge using the public CED.

Questions By Unit in CED:

Unit 1: 21, 24, 76, 90, FRQ 1(a), FRQ 5 (d)

Unit 2: 6, 8, 25, 28, 80, FRQ 5 (c), FRQ 6 (a)

Unit 3: 14, 16, 81, FRQ 4 (c), FRQ 6 (b)

Unit 4: 5, 10, 12, 18, 23, 87, 88, FRQ 1(d), FRQ 2 (b) (c), FRQ 5 (b) (c), FRQ 6 (d)

Unit 5: 9, 13, 15, 82, 83, 84, 85, FRQ 3 (b) (c)

Unit 6: 3, 4, 11, 19, 20, 22, 29, 78, 79, 86, FRQ 3 (a) (d), FRQ 5 (a) (b), FRQ 6 (c)

OMIT Unit 7: 2, 7, 27, FRQ 4 (a) (b)

Unit: 8: 1, 17, 26, 30, 77, 89, FRQ 1 (b) (c), FRQ 2 (a) (d)

Non-Calculator Multiple Choice

1) A particle moves along a straight line so that at time t $\geq$ 0 its acceleration is given by the function $a(t) = 4t$. At time t = 0, the velocity of the particle is 4 and the position of the particle is 1. Which of the following is an expression for the position of the particle at time t $\geq$ 0?

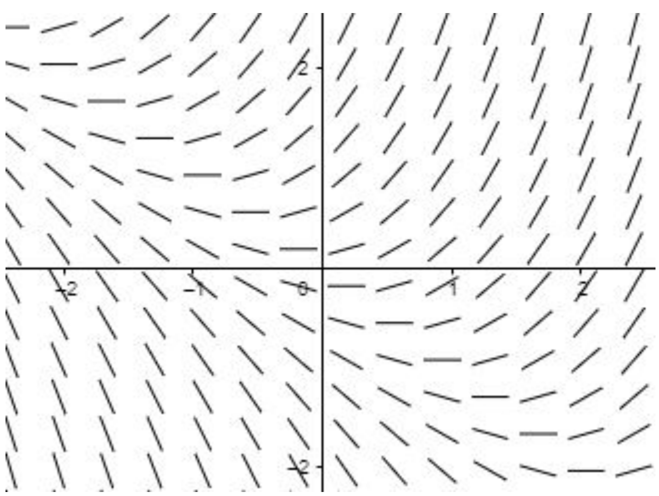
(a) $\frac{2}{3}t^3 + 4t + 1$

(b) $2t^3 + 4t + 1$

(c) $\frac{1}{3}t^3 + 4t + 1$

(d) $\frac{2}{3}t^3 + 4$

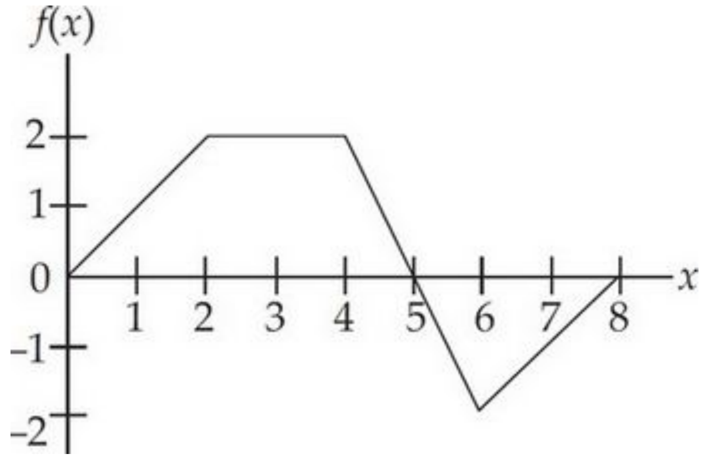
OMIT 2)



Shown above is a slope field for which of the following differential equations?

(a) $\frac{dy}{dx} = \frac{x}{y}$ (b) $\frac{dy}{dx} = xy$ (c) $\frac{dy}{dx} = x + y$ (d) $\frac{dy}{dx} = x - y$

3)



The graph of a piecewise linear function f(x) is above. Evaluate $\int_3^8 f'(x)\,dx$

(a) 2 (b) -2 (c) 5 (d) 0

4)

$$\int_1^5 \frac{x-1}{x}\,dx$$

(a) $5 - ln\ 5$ (b) $4 - ln\ 5$ (c) $2 - ln\ 5$ (d) $1 - ln\ 5$

5)

$$\lim_{x \to 1} \frac{2 \cdot \ln(x)}{e^x - 1} \text{ is}$$

(a) $\frac{2}{e}$ (b) 1 (c) 0 (d) *nonexistent*

$$f(x) = \begin{cases} x + 5 & x < -2 \\ x^2 + 2x + 3 & x \geq -2 \end{cases}$$

6) Let f be the function defined above. Which of the following statements about f is true?

(a) f is continuous and differentiable at x = -2.

(b) f is continuous but not differentiable at x = -2.

(c) f is differentiable but not continuous at x = -2.

(d) f is defined but is neither continuous nor differentiable at x = -2.

OMIT 7) The equation $y = e^{2x}$ is a particular solution to which of the following differential equations?

(a) $\frac{dy}{dx} = 1$ (b) $\frac{dy}{dx} = y$ (c) $\frac{dy}{dx} = y + 1$ (d) $\frac{dy}{dx} = y - 1$

8) For any real number x, $\lim_{h \to 0} \frac{cos((x+h)^2) - cos(x^2)}{h} =$

(a) $cos(x^2)$ (b) $2xcos(x^2)$ (c) $-sin(x^2)$ (d) $-2xsin(x^2)$

9) What is the value of x at which the maximum value of $y = \frac{4}{3}x^3 - 8x^2 + 15x$ occurs on the closed interval [0, 4]?

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 4

10) At time t = 0, a reservoir begins filling with water. For t > 0 hours, the depth of the water in the reservoir is increasing at a rate of R(t) inches per hour. Which of the following is the best interpretation of $R'(2) = 4$?

(a) The depth of the water is 4 inches, at t = 2 hours.

(b) The depth of the water is increasing at a rate of 4 inches per hour, at t = 2 hours.

(c) The rate of change of the depth of the water is increasing at a rate of 4 inches per hour per hour at t = 2 hours

(d) The depth of the water increased by 4 inches from t = 0 to t = 2 hours.

11) If $f'(x) = 3x^2$ and $f(2) = 3$, then $f(1) =$

(a) -7 (b) -4 (c) 7 (d) 10

12) A particle moves along the x-axis so that at time t $\geq$ 0 its velocity is given by $v(t) = e^{t-1} - 3sin(t-1)$. Which of the following statements describes the motion of the particle at time $t = 1$?

(a) The particle is speeding up at $t = 1$.

(b) The particle is slowing down at $t = 1$.

(c) The particle is neither speeding up nor slowing down at $t = 1$.

(d) The particle is at rest at $t = 1$.

13)

x	0	2	4	6	8
$f(x)$	1	-1	-5	7	5

The table above gives selected values for the twice-differentiable function f. In which of the following intervals must there be a number c such that

$f'(c) = -2.$

(a) $(0, 2)$ (b) $(2, 4)$ (c) $(4, 6)$ (d) $(6, 8)$

14) $\frac{d}{dx}(tan(ln(x))) =$

(a) $\dfrac{tan(ln(x))}{x}$ (b) $sec^2(ln(x))$ (c) $\dfrac{sec^2(ln(x))}{x}$ (d) $tan(\frac{1}{x})$

15) The function f is given by $f(x) = x^3 - 2x^2$. On what interval(s) is f(x) concave down?

(a) $(0, \frac{4}{3})$ (b) $(-\infty,\ 0)$ and $(\frac{4}{3}, \infty)$ (c) $(-\infty,\ \frac{2}{3})$ (d) $(\frac{2}{3}, \infty)$

16) If $\sqrt{x} + y^2 = xy + 2,$ what is $\frac{dy}{dx}$ at the point (4,0)?

(a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $-\frac{1}{4}$ (d) $\frac{1}{4}$

17) Let R be the region bounded by the graphs of $y = 2x$ and $y = x^2$. What is the area of R?

(a) 0 (b) 4 (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

18) A block of ice in the shape of a cube melts uniformly maintaining its shape. The volume of a cube given a side length is given by the formula $V = S^3$. At the moment S = 2 inches, the volume of the cube is decreasing at a rate of 5 cubic inches per minute. What is the rate of change of the side length of the cube with respect to time, in inches per minute, at the moment when S = 2 inches?

(a) $-\frac{5}{12}$ (b) $\frac{5}{12}$ (c) $-\frac{12}{5}$ (d) $\frac{12}{5}$

19) Let g be the function given by $f(x) = \int_1^x (3t - 6t^2)dt$ .What is the x-coordinate of the point of inflection of the graph of f?

(a) $-\frac{1}{4}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{1}{2}$

20) $\int sin(3x)dx =$

(a) $3cos(3x) + C$

(b) $\frac{1}{3}cos(3x) + C$

(c) $-3cos(3x) + C$

(d) $-\frac{1}{3}cos(3x) + C$

21) How many removable discontinuities does the graph of $y = \frac{x+2}{x^4+16}$ have?

(a) one (b) two (c) three (d) four

22) If $\int_{4}^{6} f(x)dx = 5$ and $\int_{10}^{4} f(x)dx = 8$ then what is the value of $\int_{6}^{10} (4f(x) + 10)dx$

(a) -12 (b) 12 (c) 52 (d) 62

23) What is the equation of the line tangent to the graph $y = e^{2x}$ at $x = 1$?

(a) $y + 2e^2 = e^2(x - 1)$

(b) $y + e^2 = 2e^2(x - 1)$

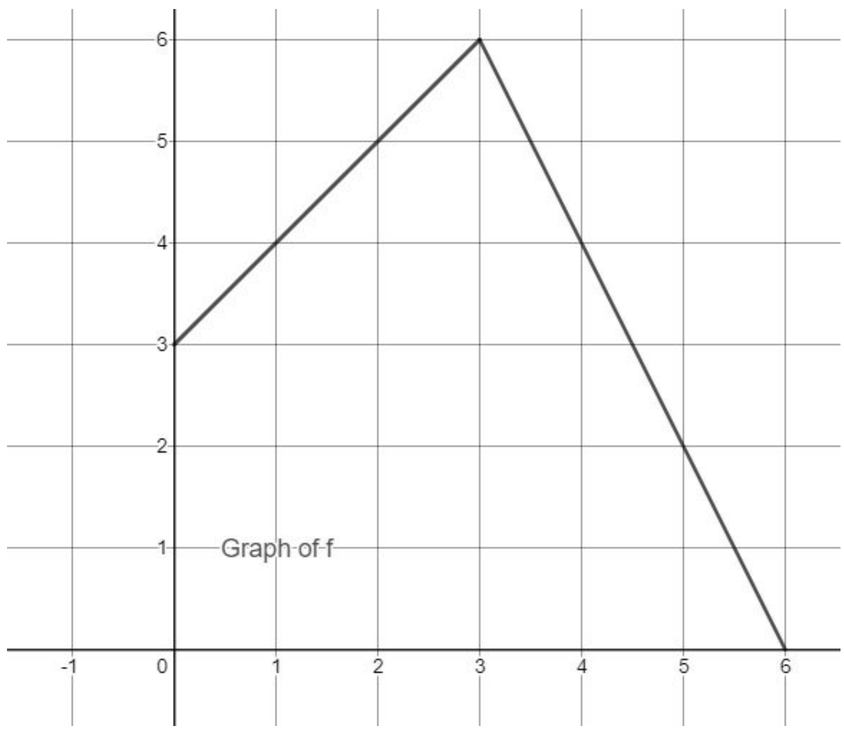
(c) $y - 2e^2 = e^2(x - 1)$

(d) $y - e^2 = 2e^2(x - 1)$

24)

$$\lim_{x\to\infty} \frac{4\cdot \ln(x)+4}{3x} =$$

(a) 2 (b) -2 (c) 0 (d) *nonexistent*



25) The graph of a function, f is shown above. Let h(x) be defined as $h(x) = (x+1)\cdot f(x)$. Find $h'(4)$.

(a) -6 (b) -2 (c) 4 (d) 14

26) A region R is the base of a solid where $f(x) \geq g(x)$ for all x $a \leq x \leq b$. For this solid, each cross section perpendicular to the x-axis are rectangles with height 5 times the base. Which of the following integrals gives the volume of this solid?

(a) $25\int_a^b (g(x)-f(x))^2 dx$

(b) $5\int_a^b (g(x)-f(x))dx$

(c) $5\int_a^b (f(x)-g(x))dx$

(d) $5\int_a^b (f(x)-g(x))^2 dx$

OMIT 27) If $\frac{dy}{dx} = \frac{x}{y}$ and if y = 4 when x = 2, then y =

(a) $\sqrt{\frac{1}{2}x^2 + 14}$

(b) $\sqrt{2x^2 + 8}$

(c) $\sqrt{x^2 + 6}$

(d) $\sqrt{x^2 + 12}$

28) If $f(x) = \dfrac{x^2+1}{3x}$ then $f'(x) =$

(a) $\dfrac{3-3x^2}{9x^2}$

(b) $\dfrac{3x^2-3}{9x^2}$

(c) $\dfrac{3x^2+3}{9x^2}$

(d) $\dfrac{2x}{3}$

29) $\int (2x+3)(x^2+3x)^4 dx =$

(a) $\frac{1}{5}(x^2+3x)^5 + C$

(b) $\frac{1}{10}(x^2+3x)^5 + C$

(c) $(x^2+3x)^5 + C$

(d) $5(x^2+3x)^5 + C$

x	1	4	6	7
f(x)	3	5	2	8
g(x)	2	1	0	5

30) Two differentiable functions, f and g have the property that $f(x) \geq g(x)$ for all real numbers and form a closed region R that is bounded from x = 1 to x = 7. Selected values of f and g are in the table above. Estimate the area between the curves f and g between x = 1 and x = 7 using a Right Riemann sum with the three sub-intervals given in the table.

(a) 13 (b) 19 (c) 21 (d) 27

Calculator Active Multiple Choice

$$f(x) = \begin{cases} -2x^2 - 7 & \text{if } x \leq 1 \\ -x^2 - 2kx & \text{if } x > 1 \end{cases}$$

76) Let f be the function defined above, where k is a constant. For what value of k, is f(x) continuous at x = 1?

(a) $-\frac{9}{2}$ (b) -4 (c) 4 (d) $\frac{9}{2}$

77) At time t, $0 < t < 2$, the velocity of a particle moving along the x-axis is given by $v(t) = e^{t^2} - 2$. What is the total distance traveled by the particle during the time interval $0 < t < 2$?

a) 12.453 (b) 13.368 (c) 51.598 (d) 53.598

78) Let f be a continuous function such that $\int_{2}^{5} f(x)\,dx = -4$ and $\int_{8}^{5} f(x)\,dx = 3$

then $\int_{8}^{2} f(x)\,dx =$

(a) -7 (b) -1 (c) 1 (d) 7

79) Let f be a twice-differentiable function defined by the differentiable function g such that $f(x) = \int_{-2}^{x} g(x)\,dx$. It is also known that g(x) is always concave up, decreasing, and positive for all real numbers. Which of the following could be false about f(x)?

(a) f(x) is concave down for all x

(b) f(x) is increasing for all x

(c) f(x) is negative for all x

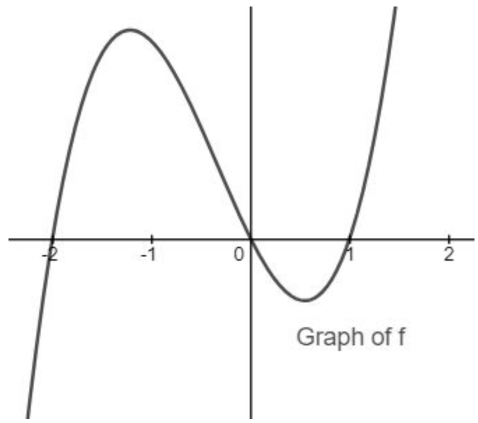
(d) f(x) = 0 for some x in the real numbers

80) Let f be the function defined by $f(x) = e^{cos(x)} - sin(x)$. For what value of x, on the interval (0,4), is the average rate of change of f(x) equal to the instantaneous rate of change of f(x) on [0,4]?

(a) 0.723 (b) 1.901 (c) 1.966 (d) 2.110

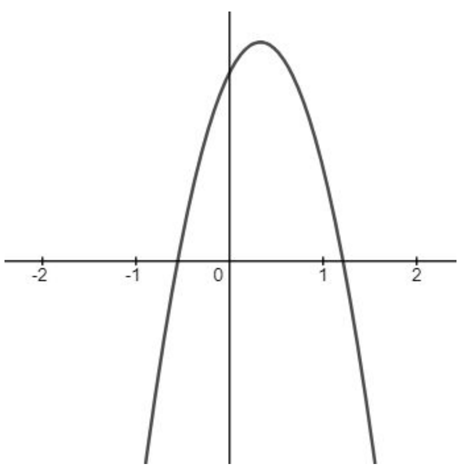
81) Let f and g be differentiable functions such that $f(g(x)) = x$ for all x. If $f(1) = 3$ and $f\,'(1) = -4$, what is the value of g'(3)?

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

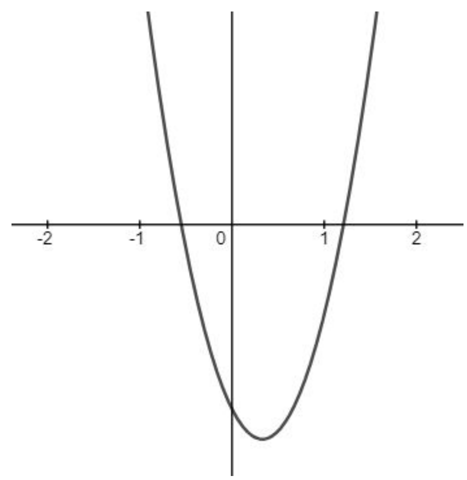


82) The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f'(x)$?

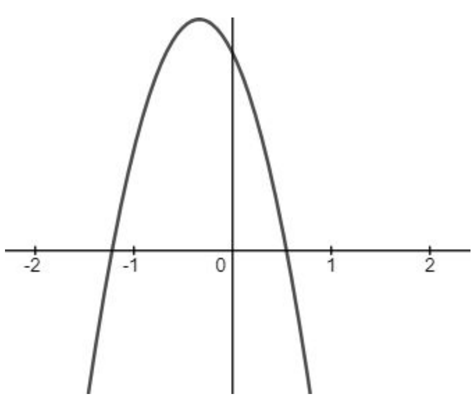
(a)



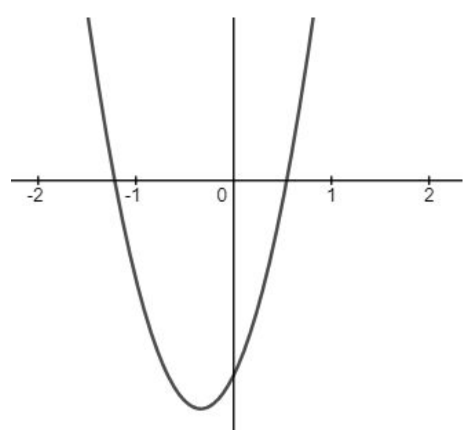
(b)



(c)



(d)



x	-5	0	3
f(x)	6	4	-2

83) The table above gives values of a differentiable function f(x) at selected x values. Based on the table, which of the following statements about f(x) could be false?

(A) There exists a value c, where $-5 < c < 3$ such that $f(c) = 1$

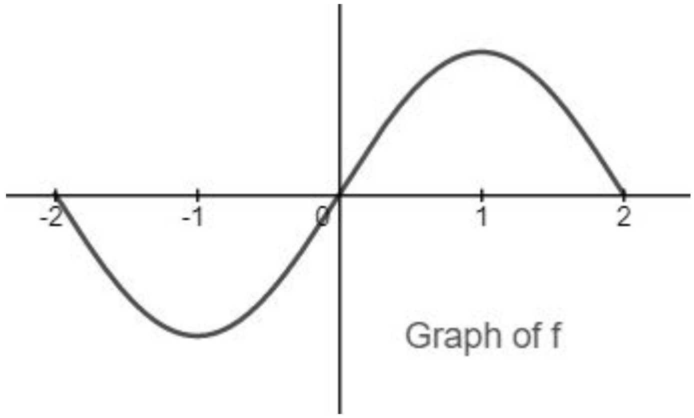
(B) There exists a value c, where $-5 < c < 3$ such that $f'(c) = 1$

(C) There exists a value c, where $-5 < c < 3$ such that $f(c) = -1$

(D) There exists a value c, where $-5 < c < 3$ such that $f'(c) = -1$

84) The function f is the antiderivative of the function g defined by $g(x) = e^x - ln(x) - 2x^2$. Which of the following is the x-coordinate of location of a relative maximum for the graph of y = f(x).

(a) 1.312 (b) 2.242 (c) 2.851 (d) 2.970



85) The function f is continuous on the closed interval [-2,2]. The graph of f', the derivative of f, is shown above. On which interval(s) is f(x) increasing?

(a) [-1,1]

(b) [-2, -1] and [1, 2]

(c) [0, 2]

(d) [-2, 0]

86) Let f be the function with the first derivative $f'(x) = \sqrt{sin(x) + cos(x) + 2}$. If g(3) = 4, what is the value of g(6)?

(a) 2.328 (b) 3.918 (c) 6.328 (d) 7.918

87) The velocity of a particle for $t \geq 0$ is given by $v(t) = ln(t^3 + 1)$. What is the acceleration of the particle at t = 4 ?

(a) 0.738 (b) 3.436 (c) 4.174 (d) 8.232

88) The function f is differentiable and f(4) = 3 and f′(4) = 2. What is the approximation of f(4.1) using the tangent line to the graph of f at x = 4 ?

(a) 2.6 (b) 2.8 (c) 3.2 (d) 3.4

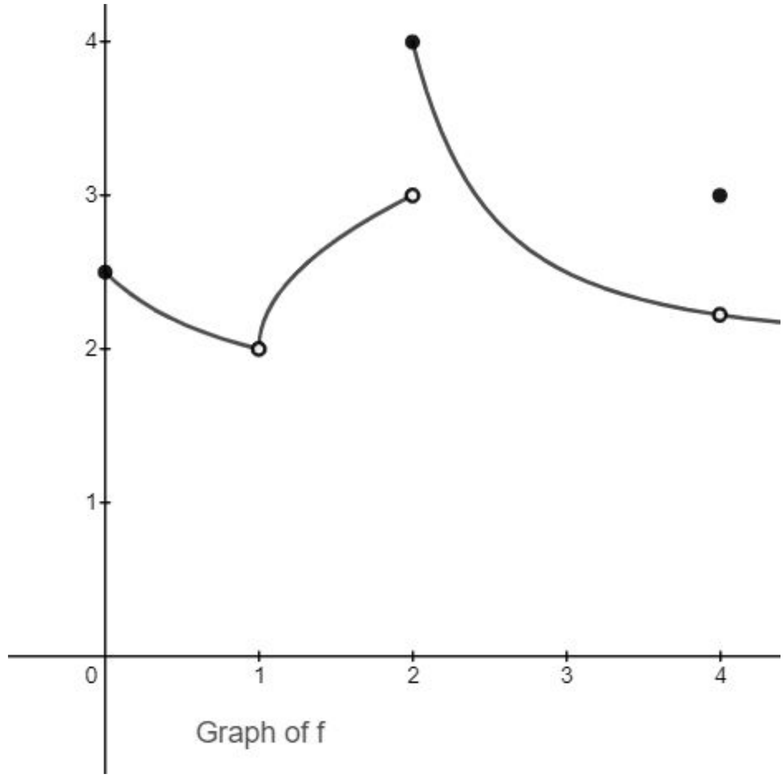
89) Patrick is climbing stairs and the rate of stair climbing is given by the differentiable function s, where s(t) is measured in stairs per second and t is measured in seconds. Which of the following expressions gives Patrick's average rate of stairs climbed from t = 0 to t = 20 seconds?

(a) $\int_0^{20} s(t)dt$

(b) $\frac{1}{20}\int_0^{20} s(t)dt$

(c) $\int_0^{20} s'(t)dt$

(d) $\frac{1}{20}\int_0^{20} s'(t)dt$



90) The graph of f is shown above. Which of the following statements is false?

(a) $f(1) = \lim_{x \to 1^-} f(x)$

(b) $f(3) = \lim_{x \to 3} f(x)$

(c) f(x) has a jump discontinuity at x = 2

(d) f(x) has a removable discontinuity at x = 4

Free Response Question with Calculator

1) A water bottle has a height of 18 centimeters and has circular cross sections. The radius, in centimeters, of a circular cross section of the bottle at height *h* centimeters is given by the piecewise function:

$$R(h) = \begin{cases} 3 & 0 \le h < 12 \\ 3 - \frac{1}{13}(h-12)^2 & 12 \le h \le 18 \end{cases}$$

(a) Is R(h) continuous at h = 12? Justify your response.

(b) Find the average value of the radius from h = 12 to h = 18.

(c) Find the volume of the water bottle. Include units.

(d) The water bottle is being filled up at a hydration station. At the instant when the height of the liquid is $h = 14$ centimeters, the height is increasing at a rate of $\frac{3}{4}$ centimeters per second. At this instant, what is the rate of change of the radius of the cross section of the liquid with respect to time?

2) Coal is burning in a furnace, thus exhausting the resource. The rate at which coal is burning, measured in pounds per hour, is given by $B(t) = 4sin(\frac{t}{2})$. At t = 2 hours, a worker starts supplying additional coal into the furnace. The rate at which coal is being added, measured in pounds per hour, is given by $S(t) = 12 + 10sin(\frac{4\pi t}{25})$. The worker stops adding coal at t = 6 hours. At t = 0 there are 500 pounds of coal in the furnace.

(a) Find the total amount of coal added by the worker. Include units.

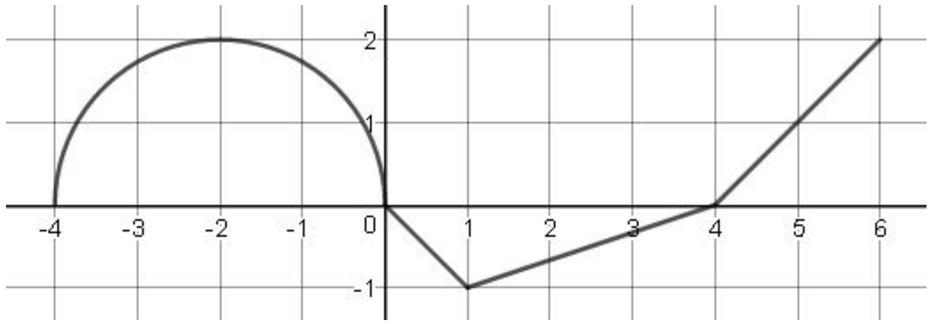
(b) Is the amount of coal in the furnace increasing or decreasing at t = 5 hours? Explain.

(c) Find S'(4). Explain, with units, the meaning of this in the context of the problem.

(d) Find the amount of coal, in pounds, in the furnace at $t = 6$ hours.

Free Response Question without Calculator

3)



Graph of f ’

The figure above represents the function f ’ the derivative of f over the interval [-4, 6] and satisfies f(4) = 2. The graph of f ’ consists of three line segments and a semi-circle.

(a) Find the value of f(-4).

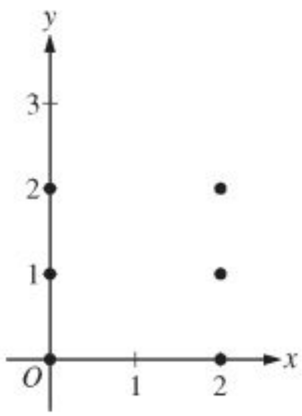
(b) On what interval(s) is f decreasing and concave up? Justify your answer.

(c) State all x-values where f(x) has a horizontal tangent on the open interval (-4, 6). Explain whether f has a relative minimum, relative maximum, or neither at each of those x-values.

(d) Evaluate $\int_{2}^{3} f''(2x)dx$

OMIT 4) Consider the differential equation, the derivative of f(x), $\frac{dy}{dx} = \frac{2y^2}{x-1}$ and where f(2) = 1.

(a) On the axes below, sketch a slope field for the given differential equation at the six points indicated.



(b) Find an expression for f(x) given that f(2) = 1.

(c) Find $\frac{d^2y}{dx^2}$

x	1	4	6	9
f(x)	10	3	5	2

5) Let g(x) be a twice-differentiable function defined by a differentiable function f, such that $g(x) = 2x + \int_{1}^{x^2} f(t)dt$. Selected values of f(x) are given in the table above.

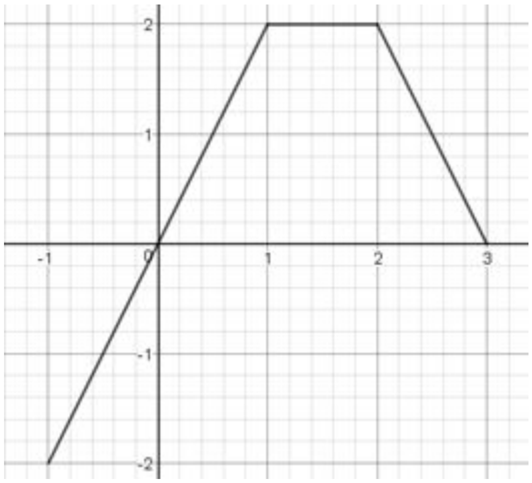
(a) Use a Left Riemann sum using the subintervals indicated by the table to approximate g(3).

(b) Find g'(x) and evaluate g'(3).

(c) Using the data in the table, estimate f'(3).

(d) Explain why there must be a value of f, on 1 < x < 9 such that $f(c) = 4$

6)



Graph of f(x)

x	g(x)	g'(x)
0	2	-4
1	1	-2
5/2	4	-3
5	-1	3

Let f be a continuous function defined on the interval [-1, 3], and whose graph is given above. Let g be a differentiable function with derivative g'. The table above gives the value of g(x) and g'(x) at selected x-values.

(a) Let h be the function defined by $h(x) = f(x) \cdot g(x)$. Find $h'(0)$

(b) Let k be the function defined by $k(x) = g(f(x))$. Find the slope of the line tangent to k(x) at $x = \frac{5}{2}$.

(c) Let $m(x) = \int_1^x g'(t)dt$. Find m(5) and m'(5)

(d) Evaluate $\lim_{x \to 0} \frac{g(x) - 2}{f(2x)}$

Multiple Choice Answers

= OMIT

1) A	24) C
2) C	25) A
3) B	26) D
4) B	27) D
5) A	28) B
6) B	29) A
7) C	30) B
8) D	
9) D	76) C
10) C	77) B
11) B	78) D
12) B	79) C
13) B	80) B
14) C	81) D
15) C	82) D
16) B	83) B
17) D	84) A
18) A	85) C
19) B	86) D
20) D	87) A
21) A	88) C
22) A	89) B
23) D	90) A

Free Response Questions Solutions

1) A water bottle has a height of 18 centimeters and has circular cross sections. The radius, in centimeters, of a circular cross section of the bottle at height *h* centimeters is given by the piecewise function:

$$R(h) = \begin{cases} 3 & 0 \le h < 12 \\ 3 - \frac{1}{13}(h-12)^2 & 12 \le h \le 18 \end{cases}$$

(a) Is R(h) continuous at h = 12? Justify your response.

$\lim_{h \to 12^-} R(h) = 3$ $\qquad \lim_{h \to 12^+} R(h) = R(12) = 3$

Since $\lim_{h \to 12^-} R(h) = \lim_{h \to 12^+} R(h) = R(12) = 3,$ R(h) is continuous at h = 12.

(b) Find the average value of the radius from h = 12 to h = 18.

$$\frac{1}{6}\int_{12}^{18} 3 - \frac{1}{13}(h-12)^2 dh = 2.077\ (or\ 2.076)\ centimeters$$

(c) Find the volume of the water bottle. Include units.

$$\pi\int_{0}^{12} (3)^2 dh + \pi\int_{12}^{18} [3 - \frac{1}{13}(h-12)^2]^2 dh = 433.451\ cm^3\ (or\ 433.450)$$

(d) The water bottle is being filled up at a hydration station. At the instant when the height of the liquid is h = 14 centimeters, the height is increasing at a rate of $\frac{3}{4}$ centimeters per second. At this instant, what is the rate of change of the radius of the cross section of the liquid with respect to time?

$$R = 3 - \frac{1}{13}(h-12)^2$$
$$\frac{d}{dt}\left[R = 3 - \frac{1}{13}(h-12)^2\right]$$
$$\frac{dR}{dt} = -\frac{2}{13}(h-12) * \frac{dh}{dt}$$
$$\frac{dR}{dt} = -\frac{2}{13}(14-12) * \frac{3}{4}$$
$$\frac{dR}{dt} = -0.231\ or\ -0.230\ cm/sec$$

2) Coal is burning in a furnace, thus exhausting the resource. The rate at which coal is burning, measured

in pounds per hour, is given by $B(t) = 4sin(\frac{t}{2})$. At t = 2 hours a worker starts supplying additional coal into the furnace. The rate at which coal is being added, measured in pounds per hour, is given by $S(t) = 12 + 10sin(\frac{4\pi t}{25})$. The worker stops adding coal at t = 6 hours. At t = 0 there are 500 pounds of coal in the furnace.

(a) Find the total amount of coal added by the worker. Include units.

(a) Coal added = $\int_2^6 S(t)\,dt$ = 78.397 pounds

78.397 pounds of coal were added by the worker.

(b) Is the amount of coal in the furnace increasing or decreasing at t = 5 hours? Explain.

(b) $S(5) = 17.878 \; or \; 17.877 \frac{lbs}{hr}$

$B(5) = 2.394 \; or \; 2.393 \frac{lbs}{hr}$

$S(5) > B(5)$ therefore, the amount of coal in the furnace is increasing at t = 5 hours.

(c) Find S’(4). Explain, with units, the meaning of this in the context of the problem.

(c) S’(4) = -2.140 lbs/hr^2

S’(4) means the rate at which coal is being added to the furnace is decreasing at a rate of 2.140 lbs/hr^2

(d) Find the amount of coal, in pounds, in the furnace at t = 6 hours.

(d) Let $A(t) = 500 + + \int_2^t S\left(x\right)\,dx - \int_0^t B(x)dx \; for \; t \geq 2$

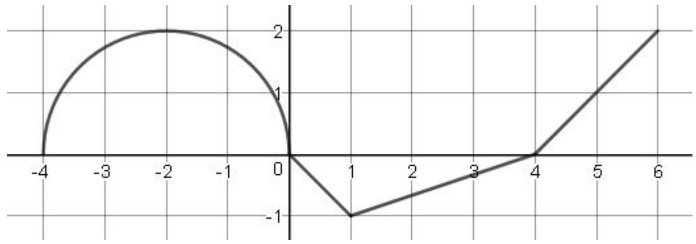
A(6) = 500 + $\int_2^6 S(t)\,dt$ - $\int_0^6 B(t)\,dt$

= 562.477 pounds

There are 562.477 pounds of coal in the furnace at t = 6 hours.

Non-Calculator Free Response

3)



Graph of f’

The figure above represents the function f’ a continuous function, the derivative of f over the interval [-4, 6] and satisfies f(0) = 4. The graph of f’ consists of three line segments and a semi-circle.

(a) Find the value of f(-4).

$$f(-4) = 4 + \int_0^{-4} f'(x)\,dx = 4 - 2\pi$$

(b) On what interval(s) is f decreasing and concave up? Justify your answer.

(b) $f(x)$ is decreasing and concave up on the interval (1,4) because f’(x) is negative and increasing.

(c) State all x-values where f(x) has a horizontal tangent on the open interval (-4, 6). Explain whether f has a relative minimum, relative maximum, or neither at each of those x-values.

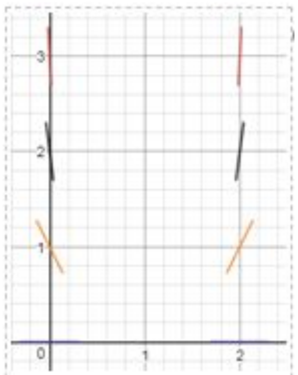
(c) At x = 0 and x =4, f(x) has a horizontal tangent.
At x = 0, f(x) has a relative maximum because f’ changes from positive to negative.
At x = 4, f(x) has a relative minimum because f’ changes from negative to positive.

(d) Evaluate $\int_2^3 f''(2x)dx$

$$\int_2^3 f''(2x)dx = \tfrac{1}{2}f'(2x)\Big|_2^3 = \tfrac{1}{2}f'(6) - \tfrac{1}{2}f'(4) = 1$$

OMIT 4) Consider the differential equation, the derivative of f(x), $\frac{dy}{dx} = \frac{2y^2}{x-1}$ and where f(2) = 1.

(a) On the axes below, sketch a slope field for the given differential equation at the six points indicated.



(b) Find an expression for f(x) given that f(2) = 1.

$\frac{1}{2y^2}dy = \frac{1}{x-1}dx$

$\int \frac{1}{2y^2}dy = \int \frac{1}{x-1}dx$

$\frac{-1}{2y} = ln|x-1| + C$

$\frac{-1}{2(1)} = ln|2-1| + C$

$C = -\frac{1}{2}$

$\frac{-1}{2y} = ln(x-1) - \frac{1}{2}$

$y = \frac{-1}{2ln(x-1)-1}$

(c) Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{\left[(x-1)\left(4y * \frac{dy}{dx}\right) - (2y^2)(1)\right]}{(x-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left[(x-1)\left(4y * \left(\frac{2y^2}{x-1}\right)\right) - (2y^2)(1)\right]}{(x-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{8y^3 - 2y^2}{(x-1)^2}$$

x	1	4	6	7
f(x)	10	8	5	2

5) Let g(x) be a twice-differentiable function defined by a differentiable function f, such that $g(x) = 2x + \int_1^{x^2} f(t)dt$. Selected values of f(x) are given in the table above.

(a) Use a Left Riemann sum using the subintervals indicated by the table to approximate g(3).

$g(3) = 2(3) + \int_1^9 f(x)dx$

$g(3) \approx 2(3) + (3)(10) + (2)(8) + (1)(5)$

$g(3) \approx 57$

(b) Find g'(3).

$g'(x) = 2 + 2x \cdot f(x^2)$

$g'(3) = 2 + 2(3)f(9)$

$g'(3) = 14$

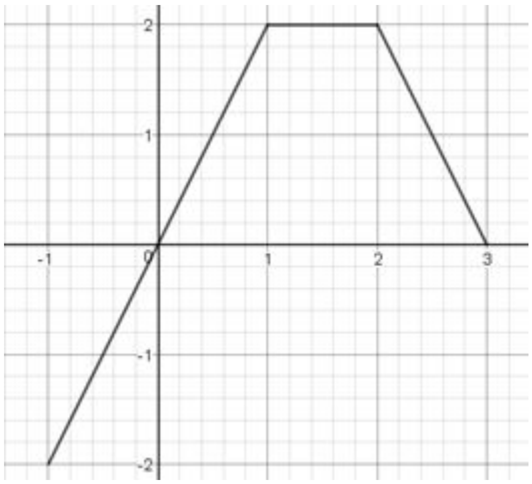
(c) Using the data in the table, estimate f'(3).

$$f'(3) \approx \frac{f(4)-f(1)}{4-1} = \frac{8-10}{3} = \frac{-2}{3}$$

(d) Explain why there must be a value of c, on 1 < x < 7 such that $f(c) = 4$

Since g(x) is twice-differentiable, g'(x) is continuous therefore IVT applies. There must be a value of c, on 1 < x < 7, such that g'(c) = f(c) = 4 because f(4) > 4 > f(6).

6)



Graph of f(x)

x	g(x)	g'(x)
0	2	-4
1	1	-2
5/2	4	-3
5	-1	3

Let f be a continuous function defined on the interval [-1, 3], and whose graph is given above. Let g be a differentiable function with derivative g'. The table above gives the value of g(x) and g'(x) at selected x-values.

(a) Let h be the function defined by $h(x) = f(x) * g(x)$. Find $h'(0)$

$h'(x) = f'(x) * g(x) + f(x) * g'(x)$

$h'(0) = f'(0) * g(0) + f(0) * g'(0)$

$h'(0) = (2) * (2) + 0 * (-4)$

$h'(0) = 4$

(b) Let k be the function defined by $k(x) = g(f(x))$. Find the slope of the line tangent to k(x) at $x = \frac{5}{2}$.

$k'(x) = g'(f(x)) * f'(x))$

$k'\left(\frac{5}{2}\right) = g'\left(f\left(\frac{5}{2}\right)\right) * f'\left(\frac{5}{2}\right)$

$k'(\frac{5}{2}) = (-3) * (-2) = 6$

(c) Let $m(x) = \int_1^x g'(t)dt$. Find m(5) and m'(5)

$$m(5) = \int_1^5 g'(t)dt$$
$$m(5) = g(5) - g(1)$$
$$m(5) = (-1) - (1)$$
$$m(5) = -2$$

$$\frac{d}{dx}\left[m(x) = \int_1^x g'(t)dt\right]$$
$$m'(x) = g'(x)$$
$$m'(5) = g'(5)$$
$$m'(5) = 3$$

(d) Evaluate $\lim_{x \to 0} \frac{g(x)-2}{f(2x)}$

$\lim_{x \to 0} g(x) - 2 = 0 \; and \; \lim_{x \to 0} f(2x) = 0$

Therefore by L'Hopital's Rule

$$\lim_{x \to 0} \frac{g(x)-2}{f(2x)} = \lim_{x \to 0} \frac{g'(x)}{2f'(2x)} = \frac{g'(0)}{2f'(0)} = \frac{-4}{2*2} = -1$$